13 RULES That Expire

Overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding can lead to misunderstanding later in studentsÕ math careers.

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Imagine the following scenario: A primary teacher presents to her students the following set of number sentences:

3 + 5 = Î
Î + 2 = 7
8 = Î + 3
2 + 4 = Î + 5.

Stop for a moment to think about which of these number sentences a student in your class would solve Þrst or Þnd easiest. What might they say about the others? In our work with young children, we have found that students feel comfortable solving the first equation because it Òlooks rightÓ and students can interpret the equal sign as Þnd the answer. However, students tend to hesitate at the remaining number sentences because they have yet to interpret and understand the equal sign as a symbol indicating a relationship between two quantities (or amounts) (Mann 2004).

In another scenario, an intermediate student is presented with the problem 43.5 10. Immediately, he responds, ÒThatÕs easy; it is 43.50 because my teacher said that when you multiply any number times ten, you just add a zero at the end.Ó

In both these situations, hints or repeated practices have pointed students in directions that are less than helpful. We suggest that these students are experiencing rules that expire. Many of these rules ÒexpireÓ when students expand their knowledge of our number systems beyond whole numbers and are forced to change their perception of what can be included in referring to a number. In this article, we present what we believe are thirteen pervasive rules that expire. We follow up with a conversation about incorrect use of mathematical language, and we present alternatives to help counteract common student misunderstandings.

The Common Core State Standards (CCSS) for Mathematical Practice advocate for students to become problem solvers who can reason, apply, justify, and effectively use appropriate mathematics vocabulary to demonstrate their understanding of mathematics concepts (CCSSI 2010). This, in fact, is quite opposite of the classroom in which the teacher does most of the talking and students are encouraged to memorize facts, Òtricks,Ó and tips to make the mathematics Òeasy.Ó The latter classroom can leave students with a collection of explicit, yet arbitrary, rules that do not link to reasoned judgment (Hersh 1997) but instead to learning without thought (Boaler 2008). The purpose of this article is to outline common rules and vocabulary that teachers share and elementary school students tend to overgeneralizeÑtips and tricks that do not promote conceptual understanding, rules that ÒexpireÓ later in studentsÕ mathematics careers, or vocabulary that is not precise. As a whole, this article aligns to Standard of Mathematical Practice (SMP) 6: Attend to precision, which states that mathematically proficient students ÒÉtry to communicate precisely to others. Éuse clear deÞnitions É and É carefully formulated explanationsÉÓ (CCSSI 2010, p.7). Additionally, we emphasize two other mathematical practices: SMP 7: Look for and make use of structure when we take a look at properties of numbers; and SMP 2: Reason abstractly and quantitatively when we discuss rules about the meaning of the four operations.

A [illegible] l [illegible]

In this section, we point out rules that seem to hold true at the moment, given the content

5. State the ÒexpirationdateÓ or the point when the rule begins to fall apart for many learners. We give the expiration date in terms of grade levels as well as CCSSM content standards in which the rule no longer ÒalwaysÓ works.

T[illegible]

1. When you multiply a number by ten, just add a zero to the end of the number.

This ÒruleÓ is often taught when students are learning to multiply a whole number times ten. However, this directive is not true when multiplying decimals (e.g., 0.25 10 = 2.5, not 0.250). Although this statement may reßect a regular pattern that students identify with whole numbers, it is not generalizable to other types of numbers. Expiration date: Grade 5 (5.NBT.2).

2. Use keywords to solve word problems.

This approach is often taught throughout the elementary grades for a variety of word problems. Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside of the problem context (Clement and Bernhard 2005). Unfortunately, many keywords are common English words that can numbers in the computation. When numbers are positive whole numbers, decimals, or fractions, subtracting will result in a number that is smaller than at least one of the numbers involved in the computation. However, if the subtraction involves two negative numbers, students may notice a contradiction (e.g., Ð5 Ð (Ð8) = 3). In division, the rule is true if the numbers are positive whole numbers, for example:

$$8 \text{ Ö } 4 = 2 \text{ or } 4 \text{ Ö } 8 = \frac{1}{2}$$

However, if the numbers you are dividing are fractions, the quotient may be larger:

$$\frac{1}{4} \text{ Ö } \frac{2}{5} = \frac{5}{8}$$

Some commonly used language Òexpires Ó and should be replaced with more appr6 () T-36 () -m t uuPg-5 f T-36 ()

relationships are Þxed. For example, the relationship between 3 and 8 is always the same. To determine the relationship between two numbers, the numbers must implicitly represent a count made by using the same unit. But when units are different, these relationships change. For example, three dozen eggs is more than eight eggs, and three feet is more than eight inches. Expiration date: Grade 2 (2.MD.2).

11. The longer the number, the larger the number.

The length of a number, when working with whole numbers that differ in the number of digits, does indicate this relationship or magnitude. However, it is particularly troublesome to apply this rule to decimals (e.g., thinking that 0.273 is larger than 0.6), a misconception noted by Desmet, GrŽgoire, and Mussolin (2010). Expiration date: Grade 4 (4.NF.7).

12. Please Excuse My Dear Aunt Sally.

This phrase is typically taught when students begin solving numerical expressions involving multiple operations, with this mnemonic serving as a way of remembering the order of operations. Three issues arise with the application of this rule. First, students incorrectly believe that they should always do multiplication before division, and addition before subtraction, because of the order in which they appear in the mnemonic PEMDAS (Linchevski and Livneh 1999). Second, the order is not as strict as students are led to believe. For example, in the expression 3^2 Ð4(2 + 7) + 8 Ö4, students have options as to where they might start. In this case, they may Þrst simplify the 2 + 7 in the grouping symbol, simplify 3^2, or divide before doing any other computationÑall without affecting the outcome. Third, the P in PEMDAS suggests that parentheses are Þrst, rather than grouping symbols more generally, which would include brackets, braces, square root symbols, and the horizontal fraction bar. Expiration date: Grade 6 (6.EE.2).

13. The equal sign means Find the answer or Write the answer.

An equal sign is a relational symbol. It indicates that the two quantities on either side of it represent the same amount. It is not a signal prompting the answer through an announcement to Òdo somethingÓ (Falkner, Levi, and Carpenter 1999; Kieran 1981). In an equation, students may see an equal sign that expresses the relationship but cannot be interpreted as Find the answer. For example, in the equations below, the equal sign provides no indication of an answer. Expiration date: Grade 1 (1.OA.7).

$$6 = Î + 4$$
$$3 + x = 5 + 2x$$

E[illegible]

In addition to helping students avoid the thirteen rules that expire, we must also pay close attention to the mathematical language we use as teachers and that we allow our students to use. The language we use to discuss mathematics (see table 1) may carry with it connotations that result in misconceptions or misuses by students, many of which relate to the Thirteen Rules That Expire listed above. Using accurate